Week 2 – Applications of Oblique Triangle Trigonometry

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How Triangulation Works

- Used to determine the location of a point by measuring angles to it from two known points

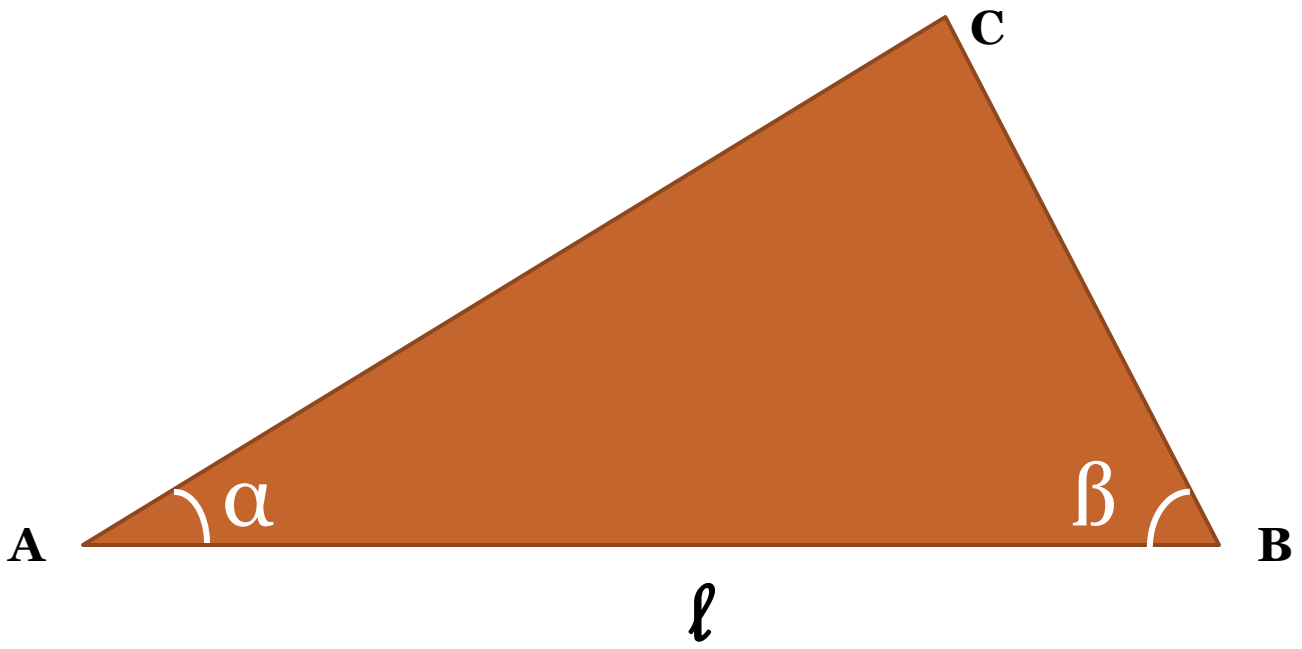


Image source: http://en.wikipedia.org/wiki/File:Domesday-book-1804x972.jpg

The Art of Surveying

- Technique of determining the terrestrial position (2D) or height (3D) by using the distances and angles between them
- Used to establish maps and boundaries for ownership
- Domesday Book completed in 1086 under William the Conqueror
  - Purpose: taxes

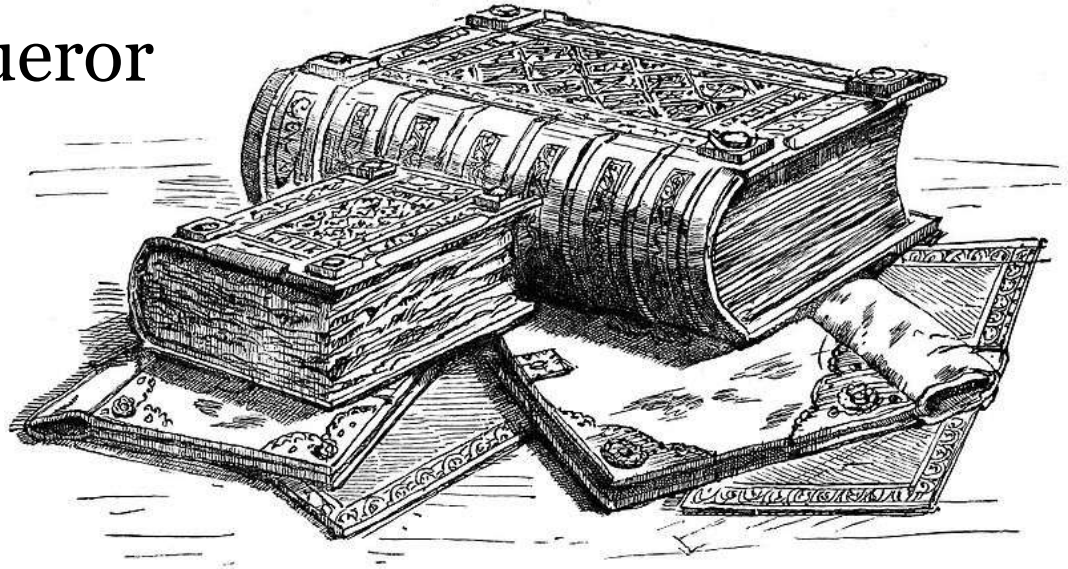
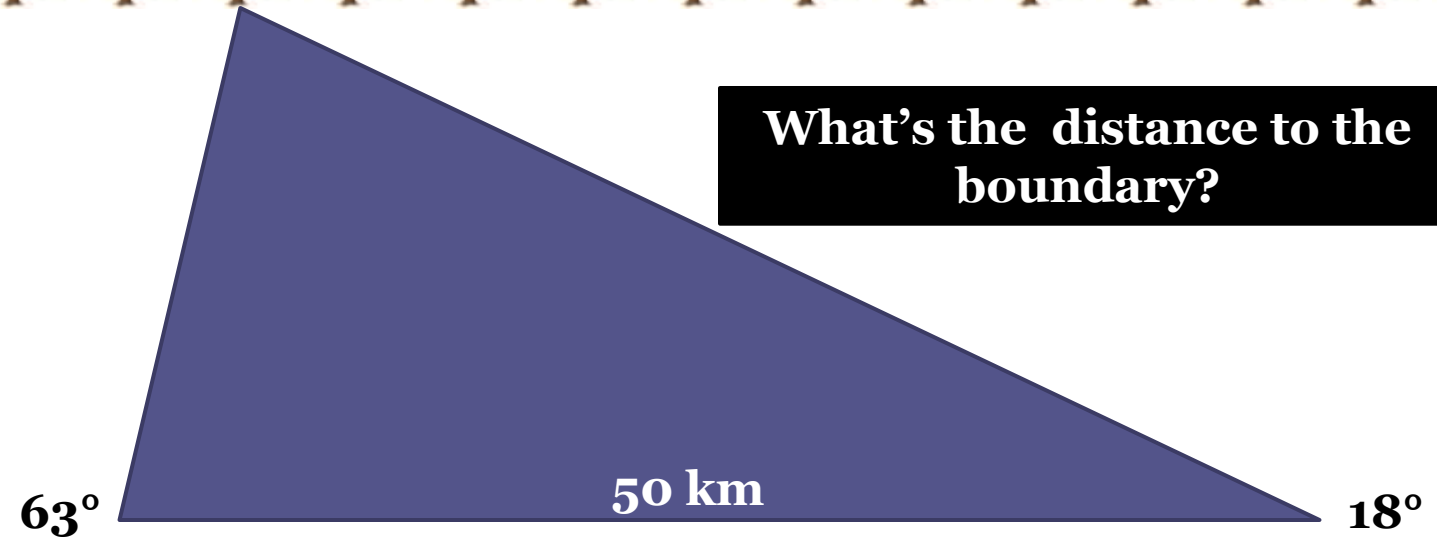


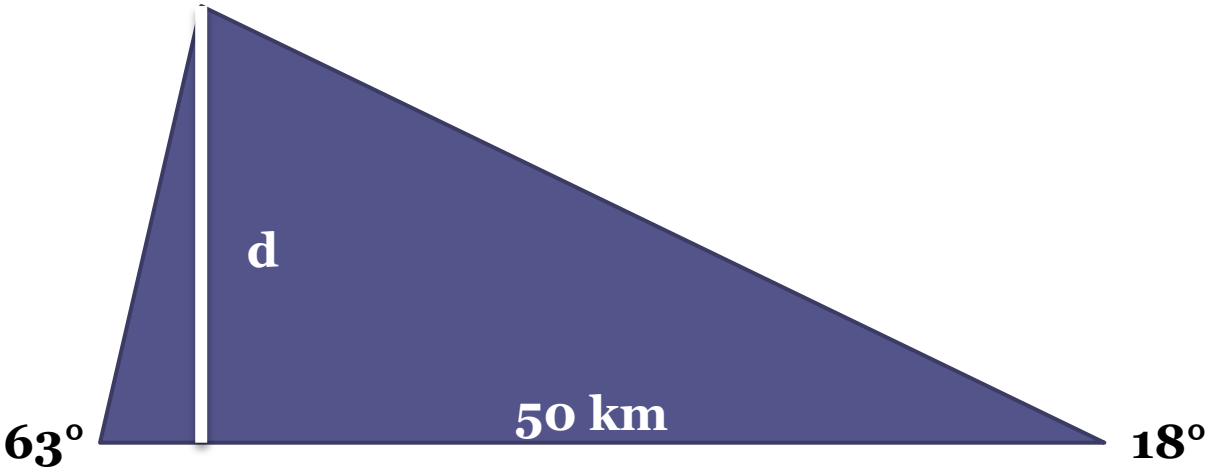
Image source: http://karenswhimsy.com/public-domain-images/tree-clipart/tree-clipart-6.jpg

Problem 1

Let's say one section of the boundary for Norfolk County is marked by a line of trees to the north.



Solution to Problem 1



$$l = \frac{d}{\tan\alpha} + \frac{d}{\tan\beta} \longrightarrow d = l \,/\, \left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$$

d = 13.9384129 so about 14 km

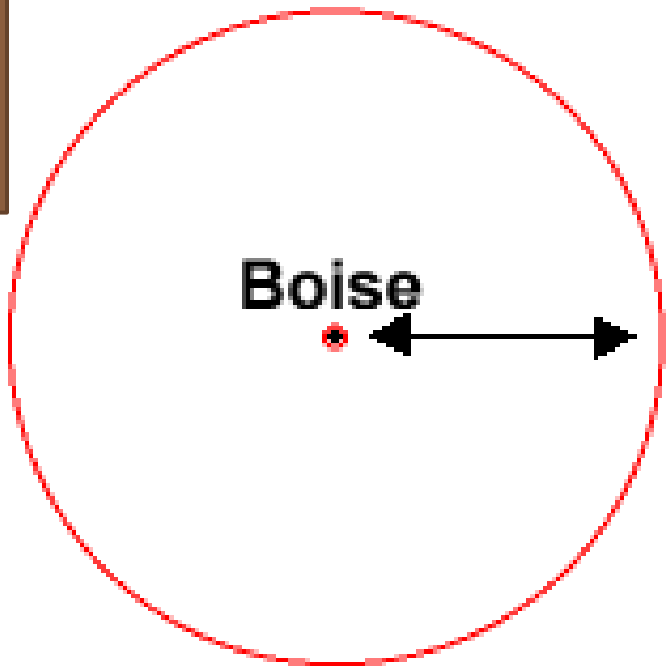
Image source: http://blog.tmcnet.com/blog/tom-keating/images/uniden-gps352.png

Modernizing Triangulation – GPS

- GPS = Global Positioning System
- 27 Earth orbiting satellites
- Originally used for military navigation system, but then opened up
- Satellites send information to receivers to determine location

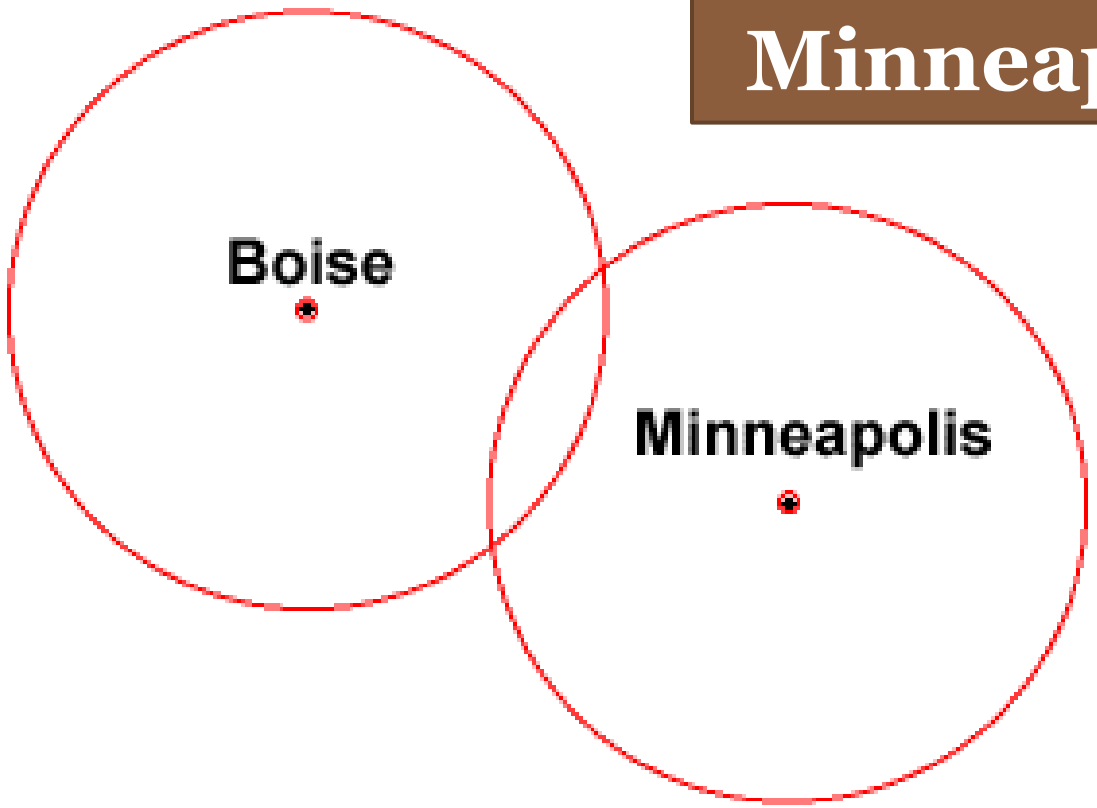
2-D Trilateriation

1006 km away from Boise

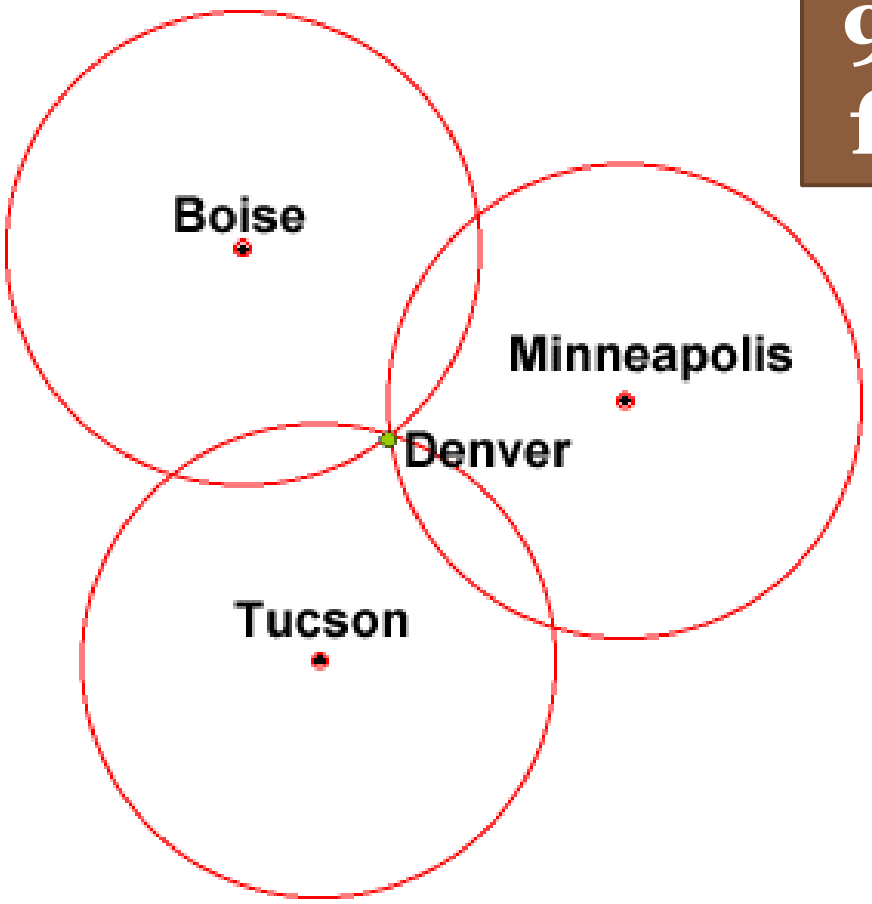


2-D Trilateration

1100 km away from Minneapolis



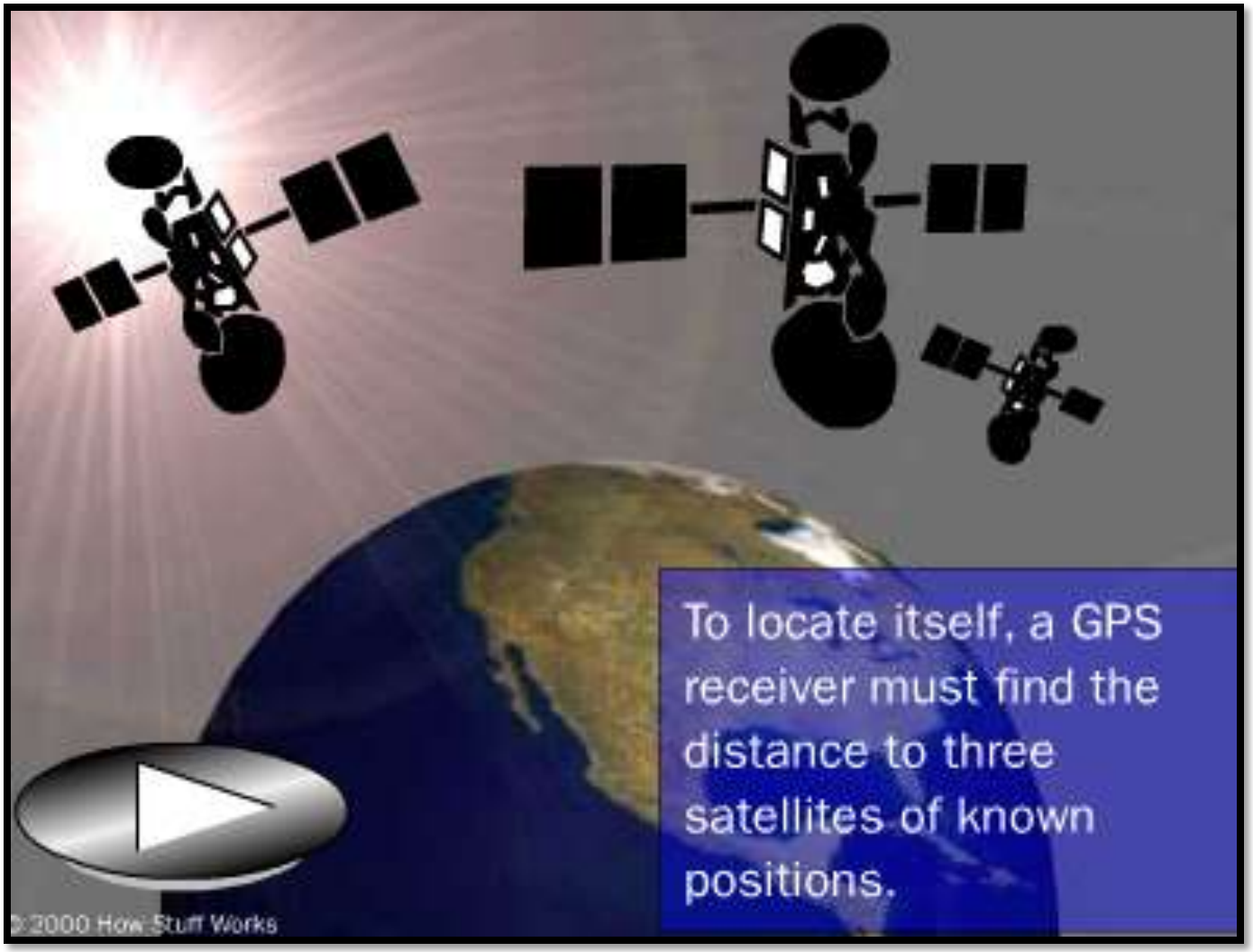
2-D Trilateration



990 km away from Tuscon

Image source: http://electronics.howstuffworks.com/gadgets/travel/gps2.htm

3-D Trilateration



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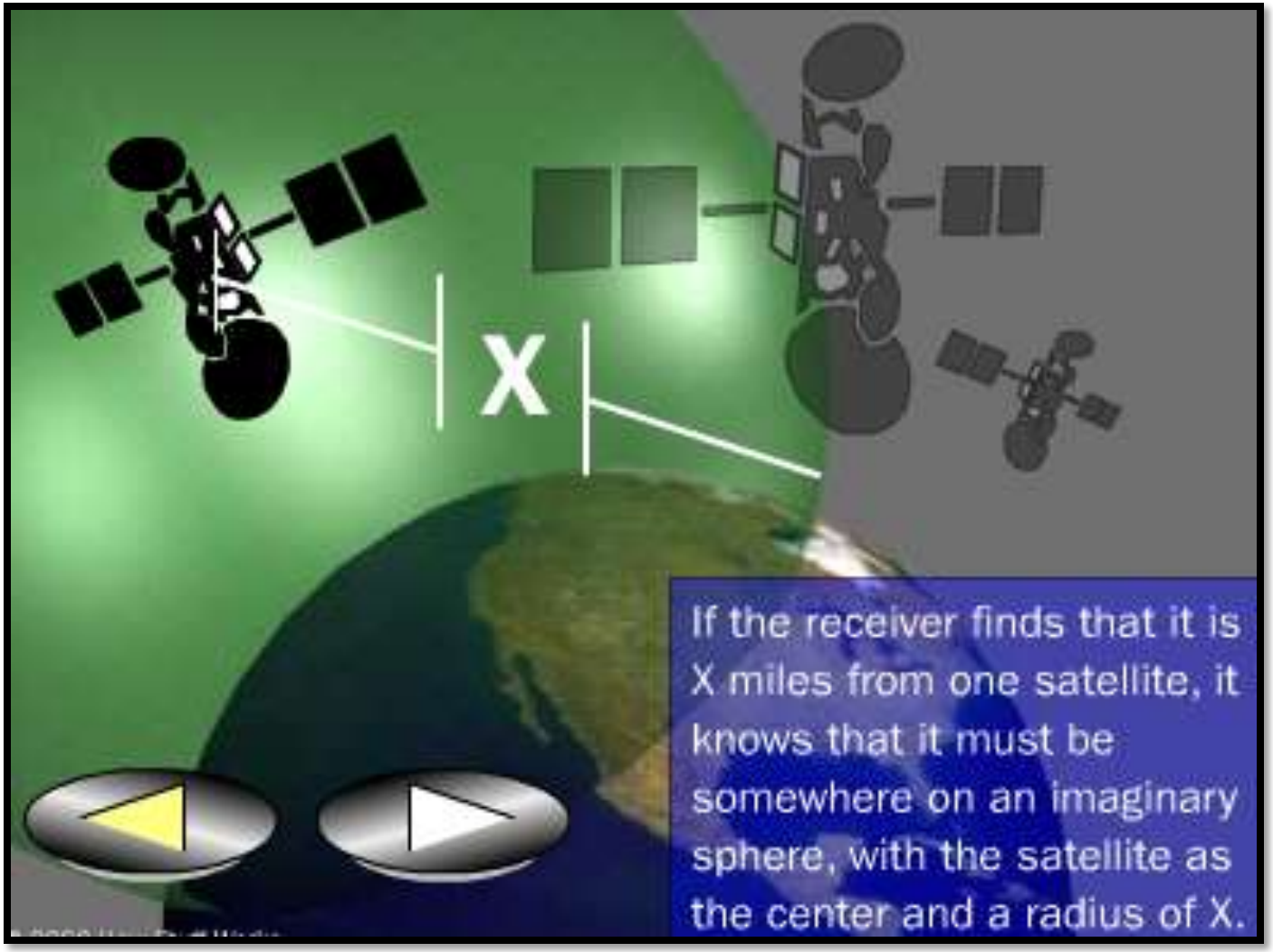
3-D Trilateration



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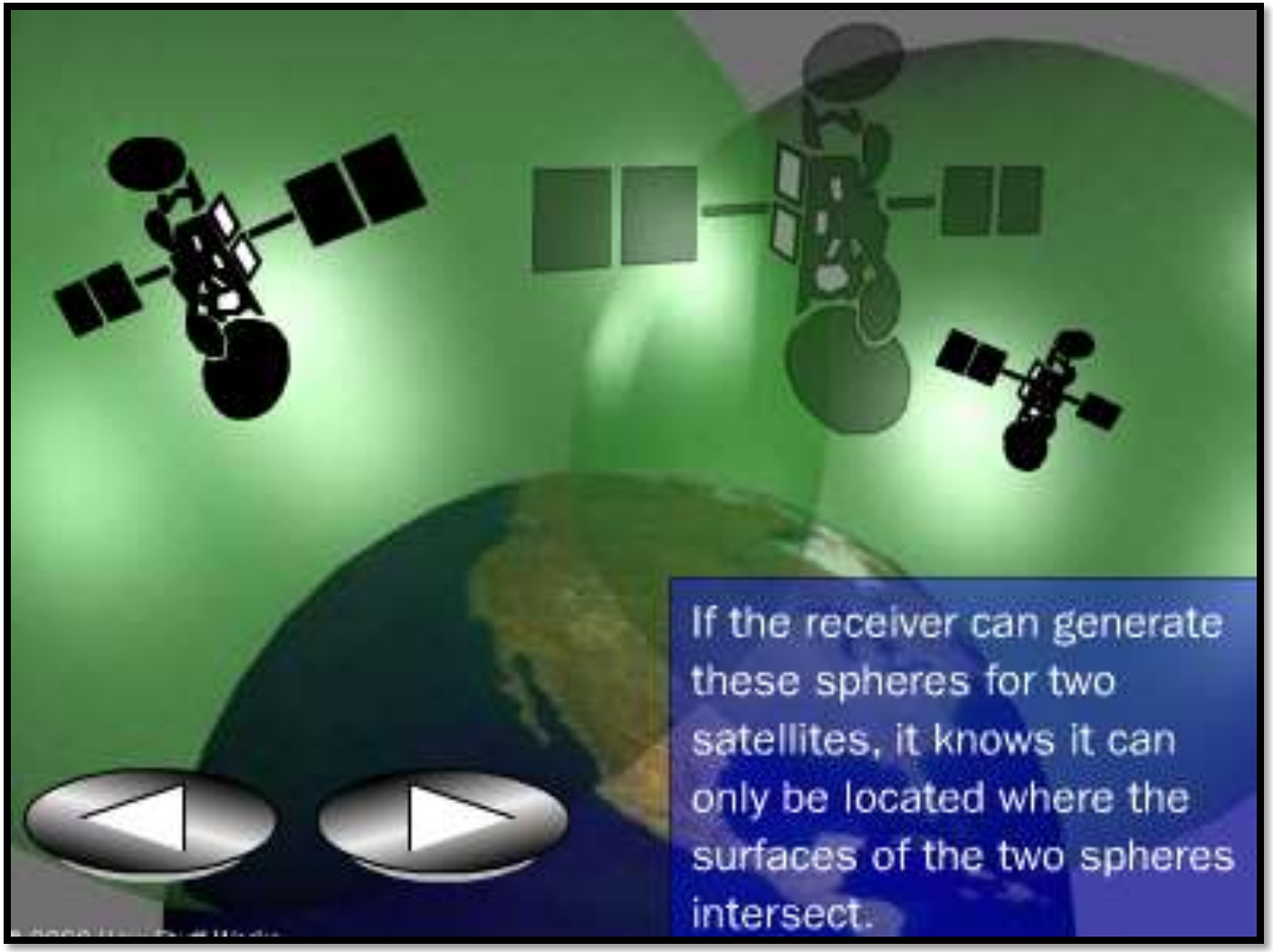
3-D Trilateration



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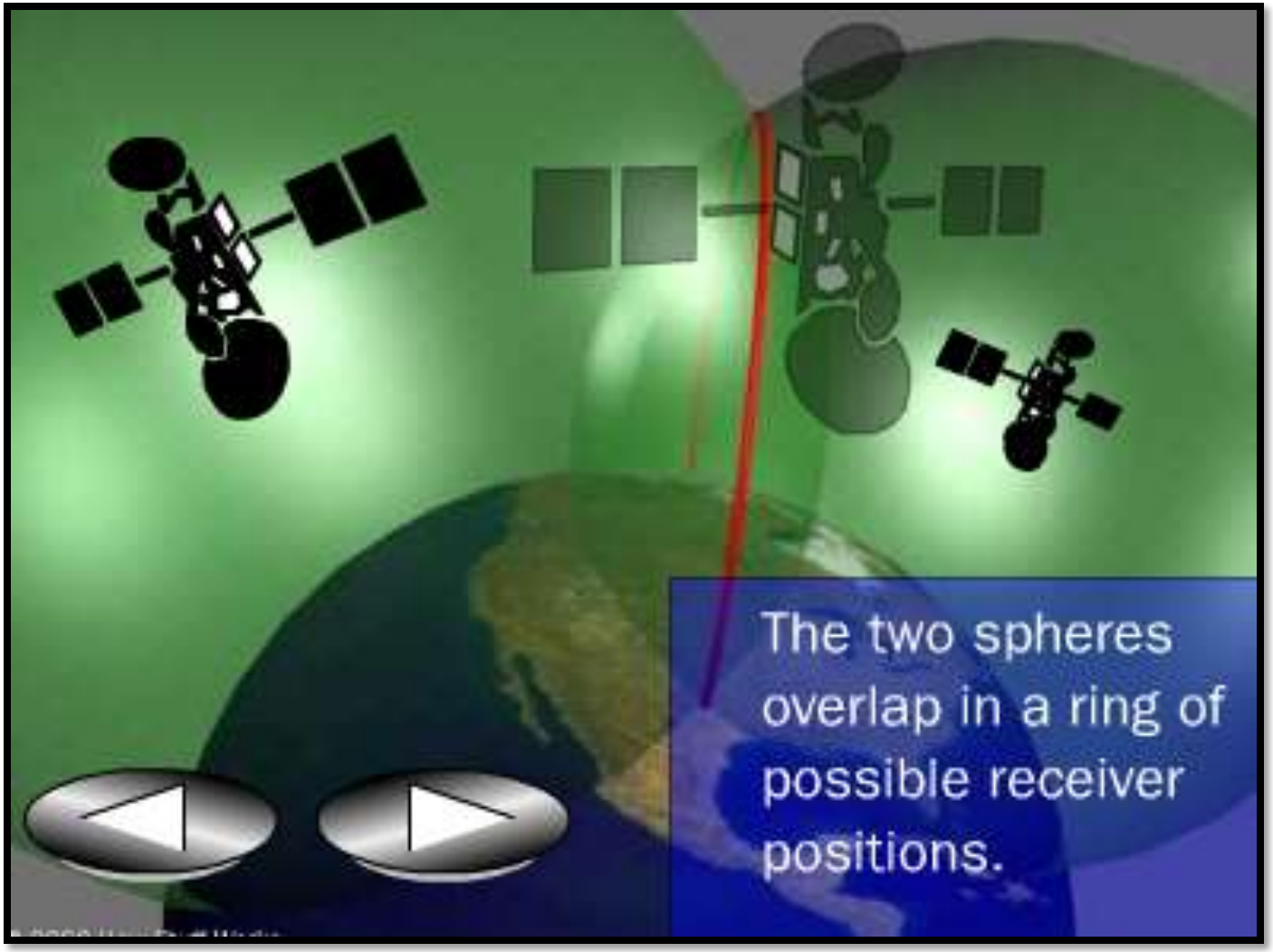

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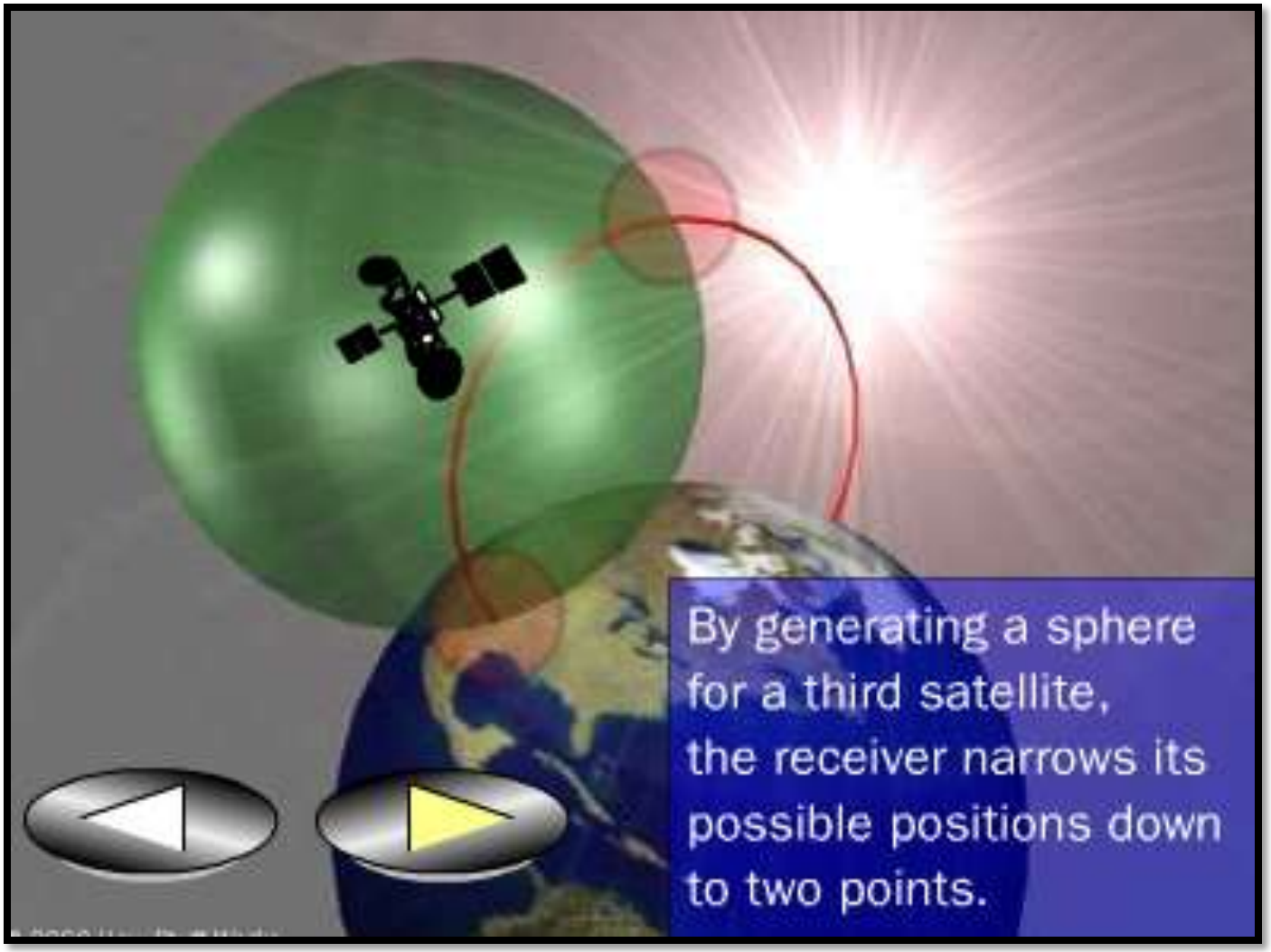
3-D Trilateration



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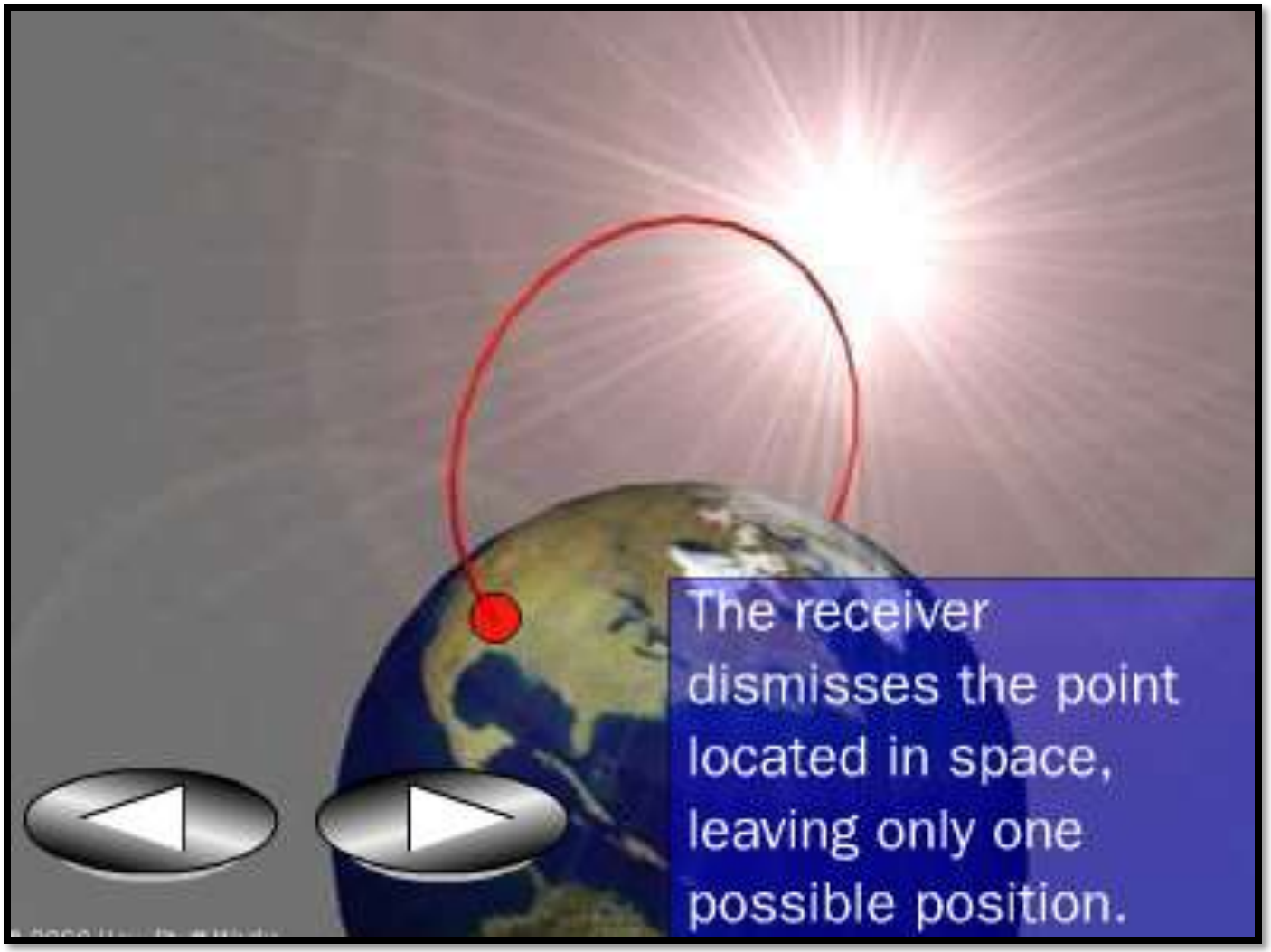
3-D Trilateration



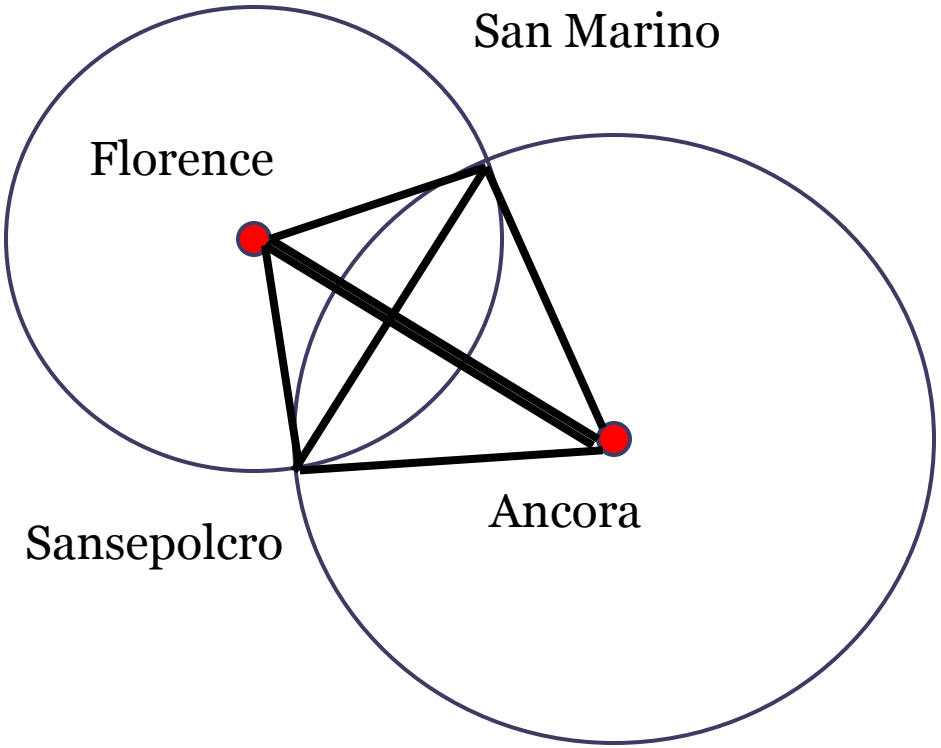
Problem 2 – Old-fashioned GPS

Your friend is lost, and their GPS is malfunctioning. He wants to know how far it is from San Marino to Sansepolcro.

1. San Marino/Sansepolcro to Florence: ~100 km
2. San Marino/Sansepolcro to Ancona: ~ 120 km
3. Florence to Ancona: 180 km

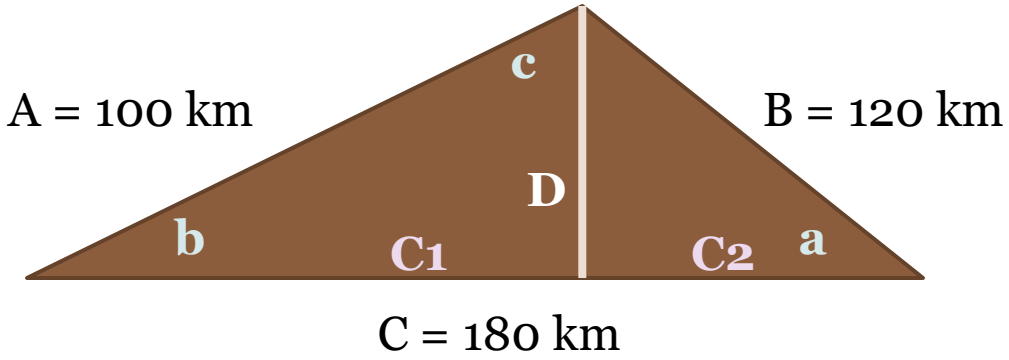
Being the good math student you are, you decide to use triangles to figure out how far your friend has to drive.

Solution to Problem 2



Use Law of Cosines

Solution to Problem 2



Use Law of Cosines: $\cos b = (A^2 + C^2 - B^2) / 2AC$

$\cos b = (100^2 + 180^2 - 120^2) / [2(100)(180)]$
$\cos b = 0.778$

$\cos b = C1/A$
$C1 = A\cos b$
$D = \text{sqrt}(A^2 - C1^2) = \text{sqrt}(A^2 - (A\cos b)^2) = A*\text{sqrt}(1-\cos^2 b) = 100*\text{sqrt}(1-0.778)$
$D = 47.11$

Multiply D by 2 to get the total distance from San Marino to Sansepolro
94 km

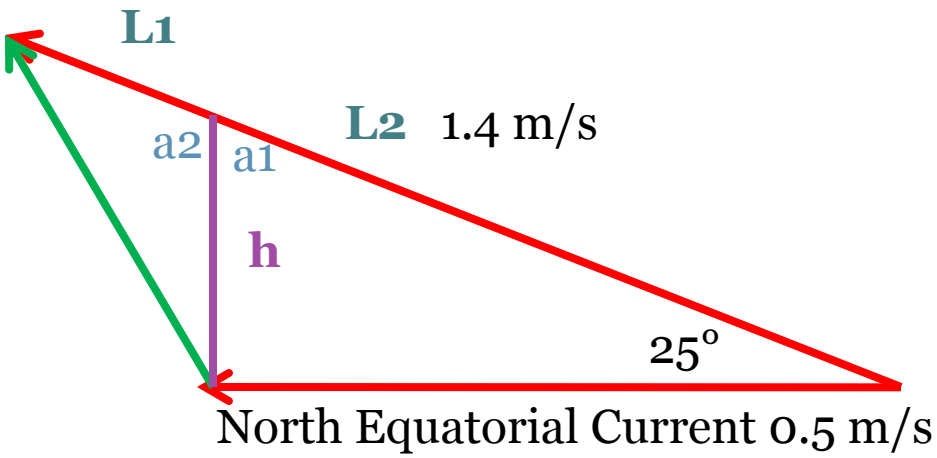
Image source: http://upload.wikimedia.org/wikipedia/commons/c/ca/Triangle_trade2.png

Triangular Trade

Problem 3

For most of the voyage from Africa to North America, the North Equatorial Current flows directly west at 0.5 m/s. If the ship wants to ultimately land in the West Indies from the Gulf of Guinea after 2 months, it has to travel at a speed of 1.4 m/s. In what direction does the ship have to leave to end up traveling 25° North of West?

Solution to Problem 3



1. Find the length of L2.

$\cos(25°) = 1/2L2$

$L2 = 1/\ 2\cos(25°) = 0.55$

2. Find the length of h

$\sin(25°) = h/L2 = 2h\cos(25°)$

$h = 2\tan(25°) = 0.93$

3. Find a2

$a2 = 180 - a1 = 180 - (90\text{-}25) = 115°$

4. Find L1

$L1 = 1.4 - L2 = 1.4 - 0.55 = 0.85$

Solve the triangle!

Solution to Problem 3

L1 = 0.85
a2 = 115°
h = 0.93

Use Law of Cosines to find the 3rd side:

$cos\ b = (A^2 + C^2 - B^2) / 2AC$

$2L1*h \cos a2 = L1^2 + h^2 - B^2$

$B^2 = L1^2 + h^2 - 2L1*h \cos a2$

$B^2 = 0.85^2 + 0.93^2 - 2*0.93*0.85 \cos 115°$

$B^2 = 2.26$

$B = 1.5$

Solution to Problem 3

Use Law of Sines to find the angle of direction:

$\sin(a2)/B = \sin(a)/L1$

$\sin(a) = (L1*\sin(a2))/B$

$a = \arcsin[(L1*\sin(a2))/B]$

$a = \arcsin[(0.85*\sin(115°))/1.5]$

a = 31° west of north